Prediction of Turbulent Boundary Layers in the Vicinity of Separation

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Abstract

IN this paper and the one that follows,[1] the discussion is limited to the case of a steady freestream, incompressible, two-dimensional mean flow over a streamlined or gently curved body or surface with a developed turbulent boundary layer upstream of the separation zone. Thus, the separation of the boundary layer is due to an adverse pressure gradient. Here we make use of the observations of Simpson et al.[2] from their recent experiments in modifying the Bradshaw et al.[3] computer prediction program for closer prediction of boundary layers near separation. The second part of this work[1] describes the prediction of the turbulent shear flowfield downstream of separation. All details including computer program listings are contained in Ref. 4.

The major differences between this work and the several previous prediction efforts are that the important normal-stress terms of the governing equations are accounted for and the location of the beginning of intermittent separation is predicted. This location is clearly important since Simpson et al.[2] observed that the time-averaged mean-pressure gradient dropped off rapidly after the beginning of backflow on an intermittent basis ($\gamma_p < 1$) or intermittent separation. Here γ_p is the fraction of time that the flow moves downstream. This first location where $\gamma_p < 1$ is where turbulent boundary-layer separation begins. Downstream where the average wall shearing stress is zero is the so-called fully-developed separation point or time-averaged separation point. It is clear[5] that with an observed Gaussian probability distribution for the instantaneous streamwise velocity $U+u$ at a point, intermittent separation occurs at the first streamwise location where $\sqrt{\overline{u^2}}/U > 1/3$. This would appear to be a good criterion for the beginning of intermittent separation. The location of fully developed separation is where $\gamma_p = 1/2$ at the wall. These data are in good agreement with the separation criteria of Sandborn as discussed by Simpson et al.[2] The point here is that the freestream pressure gradient relief appears to begin close to where intermittent separation begins. This is significant because we must know where the pressure gradient relief begins in order to properly calculate the entire flowfield. It also appears that the rapid pressure gradient relief occurs between the intermittent separation point and the fully developed separation point.

Contents

The streamwise and normal momentum equations can be combined to produce an equation with only the pressure gradient just outside the boundary layer

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\frac{-1}{\rho}\frac{\partial P_e}{dx}+\underset{\text{I}}{\frac{\partial(-\overline{uv})}{\partial y}}+\underset{\text{II}}{\nu\frac{\partial^2 U}{\partial y^2}}-\underset{\text{III}}{\frac{\partial}{\partial x}(\overline{u^2}-\overline{v^2})} \tag{1}$$

The last three terms are the Reynolds (I) and viscous (II) shearing stress gradients and the normal stresses gradient (III). In most boundary-layer cases, term III is negligible, but near separation it is about ¼ of term I in magnitude and therefore not negligible. The turbulence energy equation with terms of the same order is

$$\underset{\text{A}}{\left(\frac{U}{2}\frac{\partial\overline{q^2}}{\partial x}+\frac{V\partial\overline{q^2}}{2\partial y}\right)}+\underset{\text{B}}{\frac{\partial}{\partial y}\left(\frac{\overline{pv}}{\rho}+\frac{1}{2}\overline{q^2}v\right)}-\underset{\text{C}}{\nu\frac{\partial^2}{\partial y^2}\left(\frac{1}{2}\overline{q^2}+\overline{v^2}\right)}$$
$$+\underset{\text{D}}{\epsilon}=\underset{\text{E}}{-\overline{uv}\frac{\partial U}{\partial y}}-\underset{\text{F}'}{(\overline{u^2}-\overline{v^2})\frac{\partial U}{\partial x}} \tag{2}$$

where the terms are advection (A), turbulent diffusion (B), viscous diffusion (C), dissipation (D), shear stress production (E), and normal stresses production (F′). It is clear[2] that near separation, normal stress production can contribute up to ⅓ of the turbulence energy production and therefore is not negligible. The unmodified Bradshaw et al. method does not contain either terms III or F′. In addition to the effects explicitly contained in Eqs. (1) and (2), the normal stresses influence the three empirical functions used to relate the turbulence energy, shearing stress, diffusion, and dissipation. The normal stress influence on these functions is accounted for with the factor

$$F=1-\frac{(\overline{u^2}-\overline{v^2})\partial U/\partial x}{-\overline{uv}\partial U/\partial y} \tag{3}$$

which is the ratio of total turbulence energy production to the shear production. This approach also appears to be valid for three-dimensional separation of the turbulent boundary-layer flow.[6]

For the unseparated flow upstream of intermittent separation several different alternative models were tested for the wall region since there are several different formulations on the influence of strong adverse pressure gradients on the law-of-the-wall velocity profile. One model is that contained in the unmodified Bradshaw et al.[3] version with the McDonald-type wall law and a shearing-stress equation derived from the streamwise-momentum equation. A second model is simpler and is based on the same shearing-stress equation and the traditional law of the wall. Although the McDonald-type law supposedly accounts for pressure gradient effects, there is a great deal of experimental evidence for the traditional law of the wall. Even though $\partial\tau/\partial y = dP_e/dx$ on the wall, experimental results support the evidence of a very low shear stress gradient away from the wall for near-separation conditions. As pointed out[2,7] the normal stresses term in Eq. (1) is still significant near the wall near separation and would

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Received Jan. 28, 1977; synoptic received Sept. 26, 1977; revision received Jan. 17, 1978. Full paper available from National Technical Information Service, Springfield, Va., 22151 as N78-14311 at the standard price (available upon request).

Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Computational Methods.

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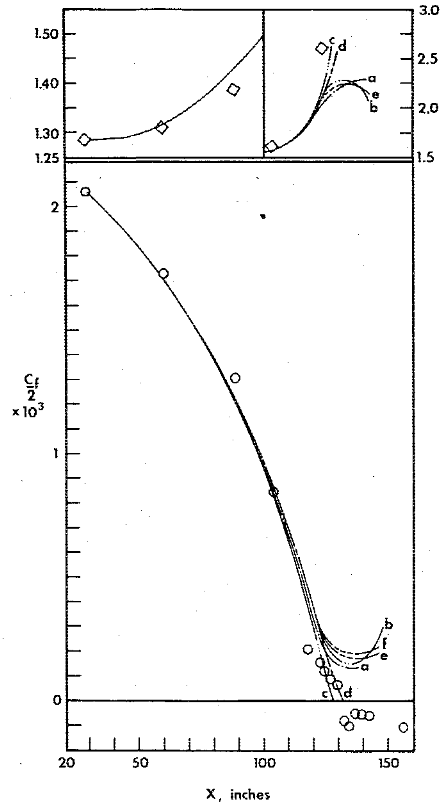

Fig. 1 Simpson et al.[2]: $C_f/2$, H, F, and δ^* experimental and predicted distributions. Flush hot film experimental skin friction values. Prediction results: a— · —model Brad, b— · · —model trad, c— · · · —model 1, d—–—model 2, e——–model 3, f———model 4.

cause the near-wall shearing-stress gradient to be lower if normal stresses were ignored. Thus with a lower shearing-stress gradient, the traditional law of the wall would be more nearly correct. These latter observations lead to third and fourth wall region models: the shearing-stress equation modified to account for the normal stresses term downstream of intermittent separation and either the McDonald law or the traditional law-of-the-wall velocity profile.

Results for two test cases are presented in Figs. 1 and 2, while Ref. 4 contains the results for several other flows. The Simpson et al.[2] flow has an airfoil type freestream velocity distribution. The Spangenberg et al.[7] flow is a Stratford type, where the boundary layer is rather abruptly subjected to a strong adverse pressure gradient that is relaxed downstream to avoid separation. Intermittent separation is properly predicted at about 121 in. in Fig. 1 with $B = U^2/u^2 = 9$ at the mesh point next to the wall. All of the modified models predict the Spangenberg et al. flow better up to intermittent separation at about 80 in. The parameter F is fairly well predicted for this flow.

Whether the traditional law-of-the-wall velocity profile or McDonald's law is used upstream of intermittent separation is not extremely important in these predictions. Upstream of the predicted location of intermittent separation for these test cases, the model 1 and model 2 predictions were within about 10% of one another. This indicates that there is only so-called weak pressure-gradient effect on the law-of-the-wall mean-velocity profile because of normal stresses. Improved predictions using model 3 or 4 were achieved in several flows but produced poorer results for the Simpson et al. flow. More work is needed on modeling the normal stress relief of the pressure gradient after the beginning of intermittent separation, but inadequate experimental data currently exist.

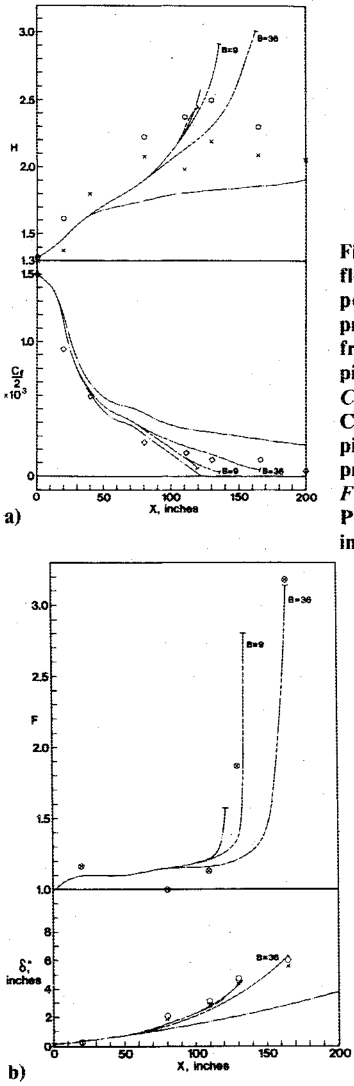

Fig. 2 Spangenberg et al.[7] flow: $C_f/2$, H, F, and δ^* experimental (flow B) and predicted distributions. Results from hot-wire data: ○, from pitot data: ×. Experimental $C_f/2$ by originators from Clauser plot of average between pitot and hot-wire velocity profiles results: ◇. Estimates of F from experimental data: ×. Predicted curves legend same as in Fig. 1.

Acknowledgments

This paper is based on work supported by the U.S. Army Research Office under grant DAHC04-75-G-0051.

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